Solutions - Homework 1

(Due date: September 21st @ 5:30 pm)

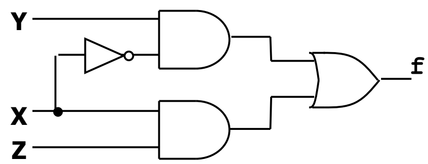
Presentation and clarity are very important!

PROBLEM 1 (25 PTS)

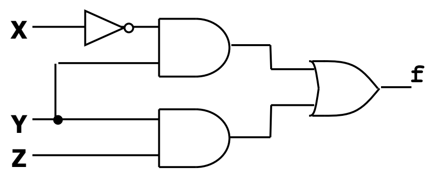
a) Simplify the following functions using ONLY Boolean Algebra Theorems. For each resulting simplified function, sketch the logic circuit using AND, OR, XOR, and NOT gates. (12 pts)

- ✓ $F(X,Y,Z) = \prod(M_0, M_1, M_4, M_6)$
- ✓ $F = (A + \bar{C} + D)(\bar{A}C + \bar{D})$
- ✓ $F = \overline{X(\bar{Y} \oplus \bar{Z}) + \bar{Y}}$
- ✓ $F = \overline{\overline{(A + B)}C + AB\bar{D}}$

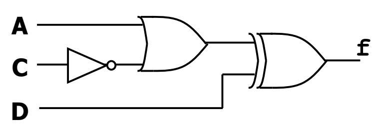
✓ $F(X,Y,Z) = \prod(M_0, M_1, M_4, M_6) = \sum(m_2, m_3, m_5, m_7) = \bar{X}Y\bar{Z} + \bar{X}YZ + X\bar{Y}Z + XYZ = \bar{X}Y(\bar{Z} + Z) + XZ(\bar{Y} + Y)$
$= \bar{X}Y + XZ$



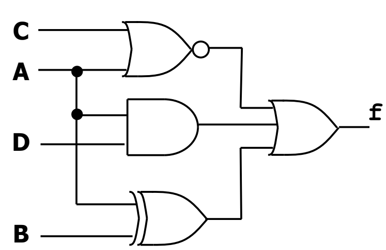
✓ $F = \overline{\overline{X(\bar{Y} \oplus \bar{Z}) + \bar{Y}}} = \overline{X(Y \oplus Z)}.Y = (\bar{X} + \overline{Y \oplus Z})Y = (\bar{X} + YZ + \bar{Y}\bar{Z})Y = \bar{X}Y + YZ$



✓ $F = (A + \bar{C} + D)(\bar{A}C + \bar{D}) = (X + D)(\bar{X} + \bar{D}) = X\bar{D} + \bar{X}D, X = A + \bar{C}$
$= (A + \bar{C})\bar{D} + \bar{A}CD = A\bar{D} + \bar{C}\bar{D} + \bar{A}CD$



✓ $F = \overline{\overline{(A + B)}C + AB\bar{D}} = \overline{\overline{(A + B)}C}.\overline{AB\bar{D}} = \overline{\bar{A}\bar{B}C}.\overline{AB\bar{D}} = (A + B + \bar{C})(\bar{A} + \bar{B} + D)$
$= A\bar{A} + A\bar{B} + AD + B\bar{A} + B\bar{B} + BD + \bar{C}\bar{A} + \bar{C}\bar{B} + \bar{C}D = A\bar{B} + \bar{A}\bar{C} + \bar{C}\bar{B} + AD + \bar{A}B + BD + \bar{C}D$
$= A\bar{B} + \bar{A}\bar{C} + AD + \bar{A}B + \bar{C}D = A\bar{B} + \bar{A}B + AD + \bar{A}\bar{C} + \bar{C}D = A\bar{B} + \bar{A}B + AD + \bar{A}\bar{C} = A \oplus B + AD + \bar{A}\bar{C}$



b) Determine whether or not the following expression is valid, i.e., whether the left- and right-hand sides represent the same function. Suggestion: complete the truth tables for both sides: (5 pts)

$$\overline{x_1}\,\overline{x_3} + x_2x_3 + x_1\overline{x_2} = \overline{x_1}x_2 + x_1x_3 + \overline{x_2}\,\overline{x_3}$$

Left-hand side:

$$\overline{x_1}(x_2+\overline{x_2})\,\overline{x_3} + (x_1+\overline{x_1})x_2x_3 + x_1\overline{x_2}(x_3+\overline{x_3}) = \overline{x_1}x_2\overline{x_3} + \overline{x_1}\,\overline{x_2}\,\overline{x_3} + x_1x_2x_3 + \overline{x_1}x_2x_3 + x_1\overline{x_2}x_3 + x_1\overline{x_2}\,\overline{x_3}$$
$$= \sum m(0,2,3,4,5,7)$$

Right-hand side:

$$\overline{x_1}x_2(x_3+\overline{x_3}) + x_1(x_2+\overline{x_2})\,x_3 + (x_1+\overline{x_1})\overline{x_2}\,\overline{x_3} = \overline{x_1}\,x_2x_3 + \overline{x_1}x_2\overline{x_3} + x_1x_2x_3 + x_1\overline{x_2}\,x_3 + x_1\overline{x_2}\,\overline{x_3} + \overline{x_1}\,\overline{x_2}\,\overline{x_3}$$
$$= \sum m(0,2,3,4,5,7)$$

Both left-hand and right-hand equations represent the same Boolean function.

c) For the following Truth table with two outputs: (8 pts)
- Provide the Boolean functions using the Canonical Sum of Products (SOP), and Product of Sums (POS).
- Express the Boolean functions using the minterms and maxterms representations.
- Sketch the logic circuits as Canonical Sum of Products and Product of Sums.

x	y	z	f_1	f_2
0	0	0	0	0
0	0	1	1	1
0	1	0	1	0
0	1	1	0	0
1	0	0	1	0
1	0	1	0	0
1	1	0	0	1
1	1	1	1	1

Sum of Products

$f_1 = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$

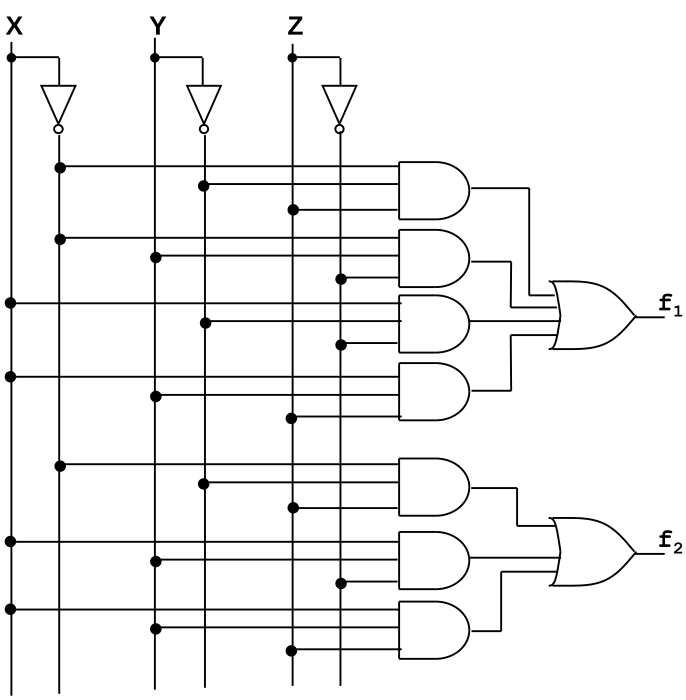
$f_2 = \bar{x}\bar{y}z + xy\bar{z} + xyz$

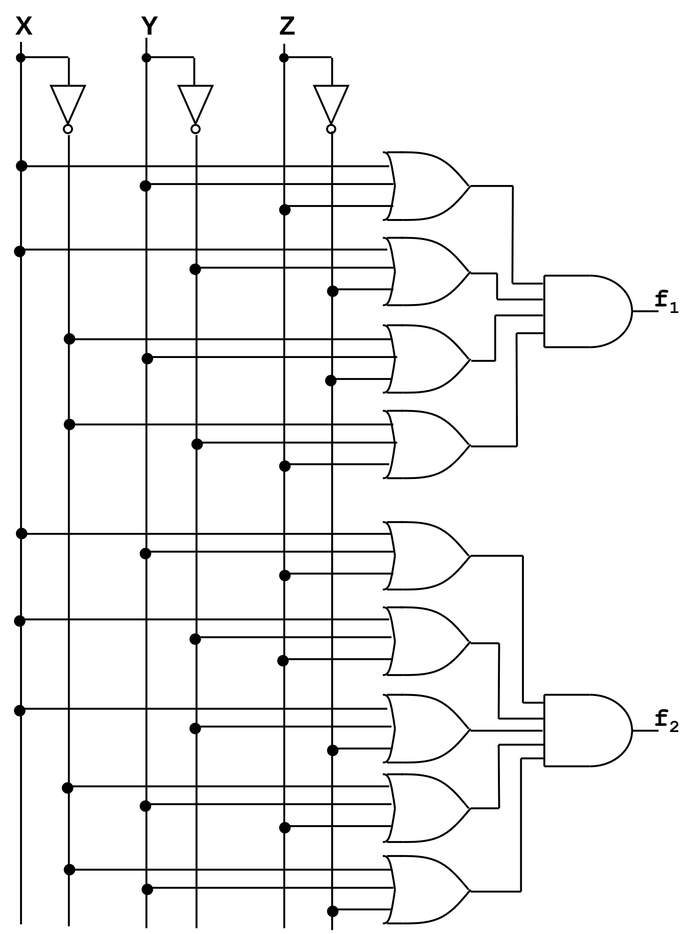
Product of Sums

$f_1 = (x+y+z)(x+\bar{y}+\bar{z})(\bar{x}+y+\bar{z})(\bar{x}+\bar{y}+z)$

$f_2 = (x+y+z)(x+\bar{y}+z)(x+\bar{y}+\bar{z})(\bar{x}+y+z)(\bar{x}+y+\bar{z})$

Minterms and maxterms: $f_1 = \sum(m_1, m_2, m_4, m_7) = \prod(M_0, M_3, M_5, M_6)$.
$f_2 = \sum(m_1, m_6, m_7) = \prod(M_0, M_2, M_3, M_4, M_5)$.

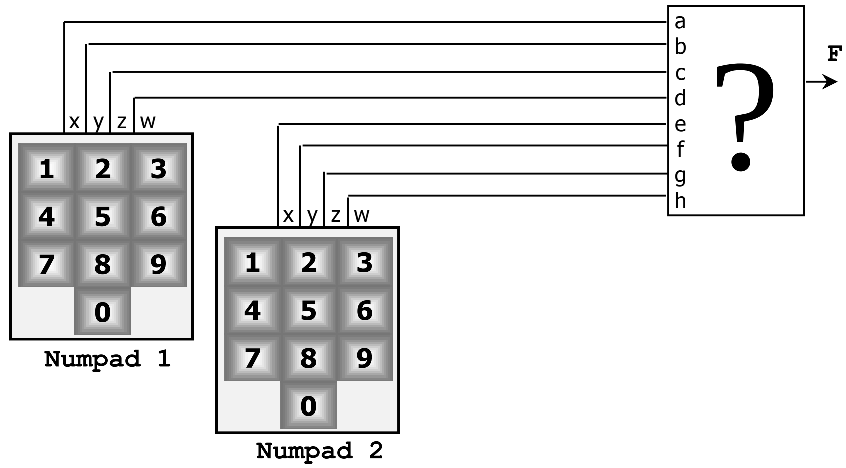




PROBLEM 2 (15 PTS)

- Design a logic circuit (<u>simplify your circuit</u>) that opens a lock ($f = 1$) whenever the user presses the correct number on each numpad (numpad 1: **8**, numpad2: **3**). The numpad encodes each decimal number using BCD encoding (see figure). We expect that the 4-bit groups generated by each numpad be in the range from `0000` to `1001`. Note that the values form `1010` to `1111` are assumed not to occur.
  <u>Suggestion</u>: Create two circuits: one that verifies the first number (**8**), and another that verifies the second number (**3**). Then perform the AND operation on the two outputs. This avoids creating a truth table with 8 inputs.
- Sketch the resulting logic circuit using ONLY 2-input NAND gates. (5 pts)

BCD code x y z w	Number pressed
0 0 0 0	0
0 0 0 1	1
0 0 1 0	2
0 0 1 1	3
0 1 0 0	4
0 1 0 1	5
0 1 1 0	6
0 1 1 1	7
1 0 0 0	8
1 0 0 1	9



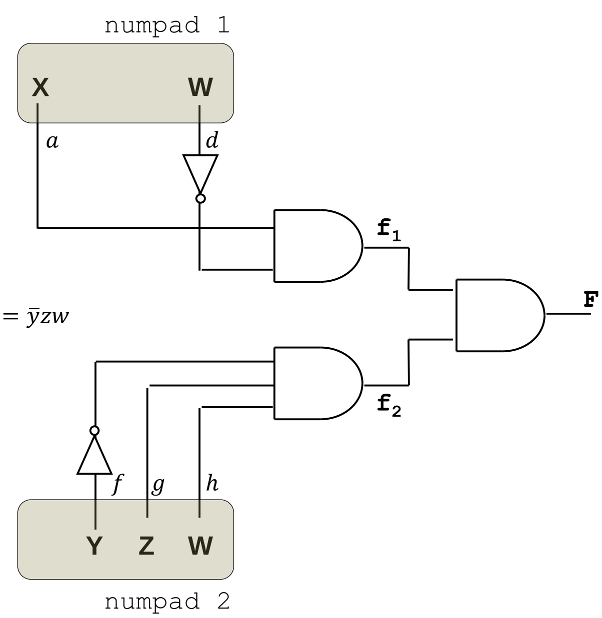
x	y	z	w	f_1	f_2
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	1
0	1	0	0	0	0
0	1	0	1	0	0
0	1	1	0	0	0
0	1	1	1	0	0
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	X	X
1	0	1	1	X	X
1	1	0	0	X	X
1	1	0	1	X	X
1	1	1	0	X	X
1	1	1	1	X	X

zw \ xy	00	01	11	10
00	0	0	X	1
01	0	0	X	0
11	0	0	X	X
10	0	0	X	X

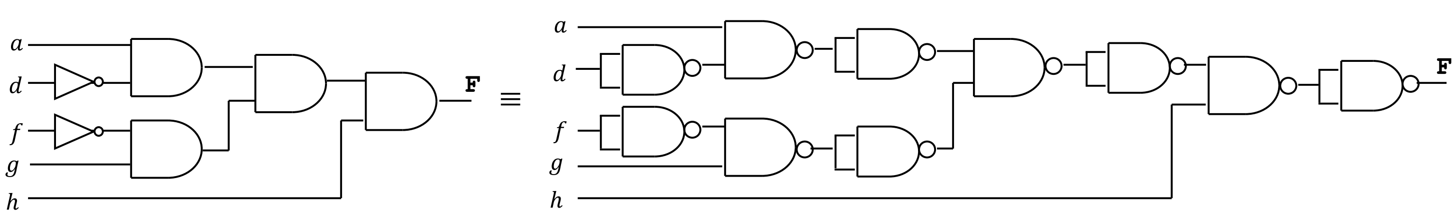
$f_1 = x\bar{w}$

zw \ xy	00	01	11	10
00	0	0	X	0
01	0	0	X	0
11	1	0	X	X
10	0	0	X	X

$f_2 = \bar{y}zw$

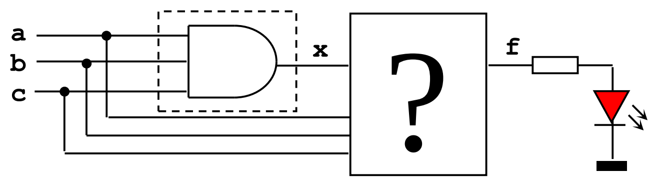


$F = a\bar{d}\bar{f}gh = (a\bar{d})(\bar{f}g)h$



PROBLEM 3 (11 PTS)

- Design a circuit (simplify your circuit) that verifies the logical operation of a 3-input AND gate. f = '1' (LED ON) if the AND gate works properly. Assumption: when the AND gate is not working, it generates 1's instead of 0's and vice versa.



x	a	b	c	f	x_{good}
0	0	0	0	1	0
0	0	0	1	1	0
0	0	1	0	1	0
0	0	1	1	1	0
0	1	0	0	1	0
0	1	0	1	1	0
0	1	1	0	1	0
0	1	1	1	0	1
1	0	0	0	0	0
1	0	0	1	0	0
1	0	1	0	0	0
1	0	1	1	0	0
1	1	0	0	0	0
1	1	0	1	0	0
1	1	1	0	0	0
1	1	1	1	1	1

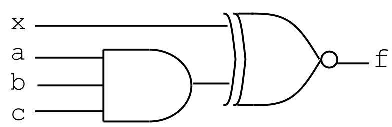
bc \ xa	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	1	0	1	0
10	1	1	0	0

$$f = \bar{x}\bar{c} + \bar{x}\bar{b} + \bar{x}\bar{a} + xabc$$

$$f = \bar{x}(\bar{a} + \bar{b} + \bar{c}) + xabc$$

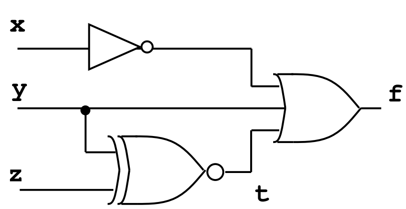
$$f = \bar{x}\overline{abc} + xabc$$

$$f = \overline{x \oplus abc}$$



PROBLEM 4 (23 PTS)

a) Construct the truth table describing the output of the following circuit and write the simplified Boolean equation (5 pts).

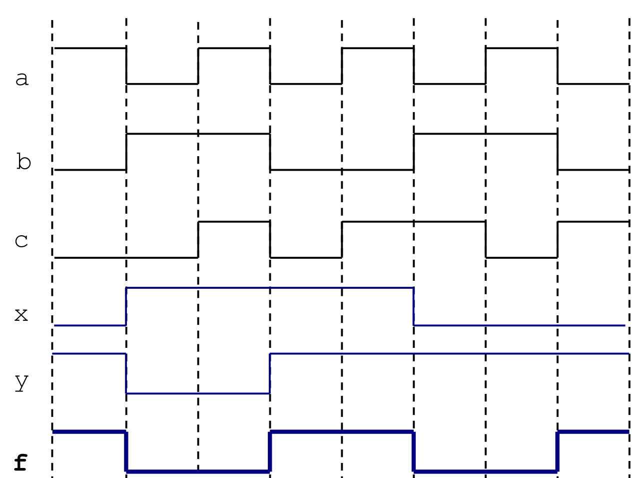


x	y	z	t	f
0	0	0	1	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	1
1	0	0	1	1
1	0	1	0	0
1	1	0	0	1
1	1	1	1	1

$f = \bar{x} + y + \bar{z}$

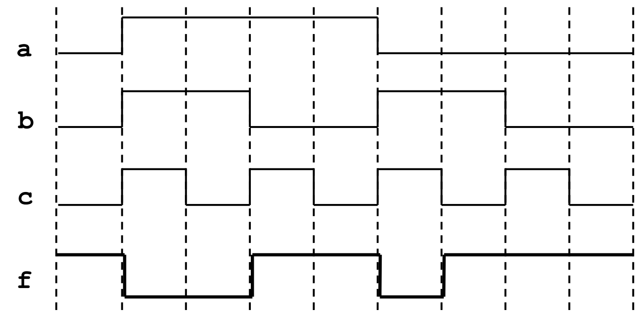
b) Complete the timing diagram of the logic circuit whose VHDL description is shown below: (5 pts)

```
library ieee;
use ieee.std_logic_1164.all;

entity circ is
  port ( a, b, c: in std_logic;
         f: out std_logic);
end circ;

architecture struct of circ is
  signal x, y: std_logic;

begin
  x <= a xnor c;
  y <= x nand b;
  f <= y and (not b);
end struct;
```



c) The following is the timing diagram of a logic circuit with 3 inputs. Sketch the logic circuit that generates this waveform. Then, complete the VHDL code. (8 pts)

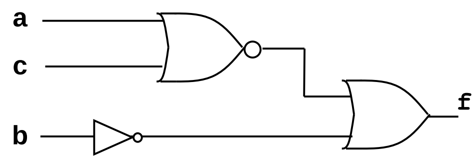
```
library ieee;
use ieee.std_logic_1164.all;

entity wav is
  port ( a, b, c: in std_logic;
             f: out std_logic);
end wav;

architecture struct of wav is

begin
   f <= not(b) or (a nor c)
end struct;
```



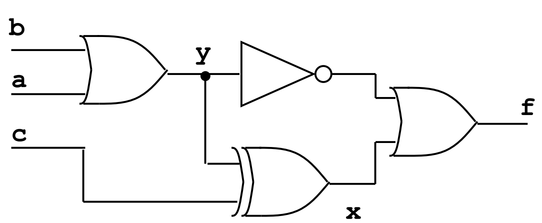
a	b	c	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

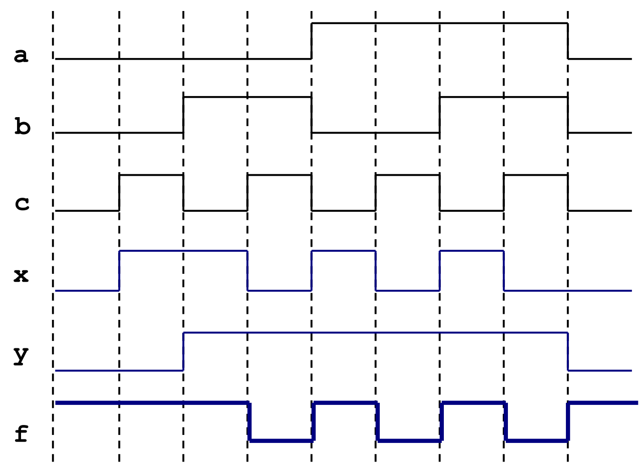
c \ ab	00	01	11	10
0	1	1	0	1
1	1	0	0	1

$f = \bar{b} + \bar{a}\bar{c} = \bar{b} + \overline{a + c}$



d) Complete the timing diagram of the following circuit: (5 pts)

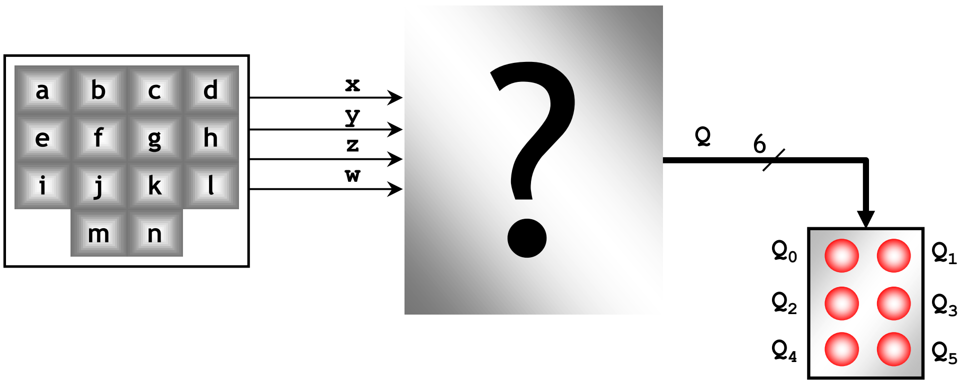




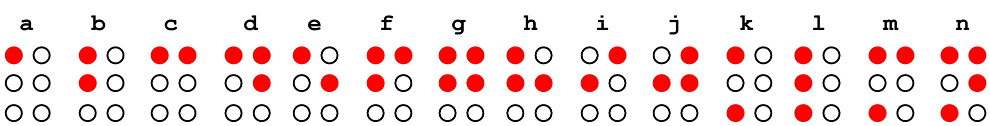
PROBLEM 5 (26 PTS)

- A 14-letter keypad produces a 4-bit code as shown in the table. We want to design a logic circuit that converts those 4-bit codes to Braille code, where the 6 dots are represented by LEDs. A raised (or embossed) dot is represented by an LED ON (logic value of '1'). A missing dot is represented by a LED off (logic value of '0').

✓ Complete the truth table for each output (Q_0-Q_5).
✓ Provide the simplified expression for each output (Q_0-Q_5). Use Karnaugh maps for Q_5,Q_4,Q_1,Q_0 and the Quine-McCluskey algorithm for Q_3-Q_2. Note it is safe to assume that the codes `1110` and `1111` will not be produced by the keypad.



x y z w	Letter
0 0 0 0	a
0 0 0 1	b
0 0 1 0	c
0 0 1 1	d
0 1 0 0	e
0 1 0 1	f
0 1 1 0	g
0 1 1 1	h
1 0 0 0	i
1 0 0 1	j
1 0 1 0	k
1 0 1 1	l
1 1 0 0	m
1 1 0 1	n
1 1 1 0	
1 1 1 1	



x y z w	Q_5	Q_4	Q_3	Q_2	Q_1	Q_0	Letter
0 0 0 0	0	0	0	0	0	1	a
0 0 0 1	0	0	0	1	0	1	b
0 0 1 0	0	0	0	0	1	1	c
0 0 1 1	0	0	1	0	1	1	d
0 1 0 0	0	0	1	0	0	1	e
0 1 0 1	0	0	0	1	1	1	f
0 1 1 0	0	0	1	1	1	1	g
0 1 1 1	0	0	1	1	0	1	h
1 0 0 0	0	0	0	1	1	0	i
1 0 0 1	0	0	1	1	1	0	j
1 0 1 0	0	1	0	0	0	1	k
1 0 1 1	0	1	0	1	0	1	l
1 1 0 0	0	1	0	0	1	1	m
1 1 0 1	0	1	1	0	1	1	n
1 1 1 0	X	X	X	X	X	X	
1 1 1 1	X	X	X	X	X	X	

Q_4

zw \ xy	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	0	X	1
10	0	0	X	1

Q_1

zw \ xy	00	01	11	10
00	0	0	1	1
01	0	1	1	1
11	1	0	X	0
10	1	1	X	0

Q_0

zw \ xy	00	01	11	10
00	1	1	1	0
01	1	1	1	0
11	0	1	X	1
10	1	1	X	1

$Q_5 = 0$
$Q_4 = xz + xy$
$Q_3 = \bar{x}y\bar{w} + \bar{x}zw + x\bar{z}w$
$Q_2 = yz + \bar{x}\bar{z}w + x\bar{y}\bar{z} + x\bar{y}w$

$Q_1 = \bar{x}\bar{y}z + y\bar{z}w + \bar{x}z\bar{w} + x\bar{z}$
$Q_0 = \bar{x} + y + z$

- $Q_3 = \sum m(3,4,6,7,9,13) + \sum d(14,15)$.

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants	1-literal implicants
1	m_4 = 0100 ✓	**$m_{4,6}$ = 01-0**	**$m_{6,14,7,15}$ = -11-** ~~$m_{6,7,14,15}$ = -11-~~	We can't combine any further, so we stop here
2	m_3 = 0011 ✓ m_6 = 0110 ✓ m_9 = 1001 ✓	**$m_{3,7}$ = 0-11** $m_{6,7}$ = 011- ✓ $m_{6,14}$ = -110 ✓ **$m_{9,13}$ = 1-01**		
3	m_7 = 0111 ✓ m_{13}= 1101 ✓ m_{14}= 1110 ✓	$m_{7,15}$ = -111 ✓ **$m_{13,15}$ = 11-1** $m_{14,15}$ = 111- ✓		
4	m_{15}= 1111 ✓			

Prime Implicants		Minterms					
		3	4	6	7	9	13
$m_{4,6}$	$\bar{x}y\bar{w}$		**X**	X			
$m_{3,7}$	$\bar{x}zw$	**X**			X		
$m_{9,13}$	$x\bar{z}w$					**X**	X
$m_{13,15}$	xyw						X
$m_{6,14,7,15}$	yz			X	X		

$$\rightarrow Q_3 = \bar{x}y\bar{w} + \bar{x}zw + x\bar{z}w$$

- $Q_2 = \sum m(1,5,6,7,8,9,11) + \sum d(14,15)$.

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants	1-literal implicants
1	m_1 = 0001 ✓ m_8 = 1000 ✓	**$m_{1,5}$ = 0-01** **$m_{1,9}$ = -001** **$m_{8,9}$ = 100-**	**$m_{6,7,14,15}$ = -11-** ~~$m_{6,14,7,15}$ = -11-~~	We can't combine any further, so we stop here
2	m_5 = 0101 ✓ m_6 = 0110 ✓ m_9 = 1001 ✓	**$m_{5,7}$ = 01-1** $m_{6,7}$ = 011- ✓ $m_{6,14}$ = -110 ✓ **$m_{9,11}$ = 10-1**		
3	m_7 = 0111 ✓ m_{11}= 1011 ✓ m_{14}= 1110 ✓	$m_{7,15}$ = -111 ✓ **$m_{11,15}$ = 1-11** $m_{14,15}$ = 111- ✓		
4	m_{15}= 1111 ✓			

Prime Implicants		Minterms						
		1	5	6	7	8	9	11
$m_{1,5}$	$\bar{x}\bar{z}w$	**X**	X					
$m_{1,9}$	$\bar{y}\bar{z}w$	X					X	
$m_{8,9}$	$x\bar{y}\bar{z}$					**X**	X	
$m_{5,7}$	$\bar{x}yw$		X		X			
$m_{9,11}$	$x\bar{y}w$						X	**X**
$m_{11,15}$	xzw							X
$m_{6,7,14,15}$	yz			**X**	X			

$$\rightarrow Q_2 = yz + \bar{x}\bar{z}w + x\bar{y}\bar{z} + x\bar{y}w$$